Burlington County Institute of Technology

Medford Campus

Westampton Campus

Full Year Algebra I Curriculum

Department: Mathematics
Credits: 10
Revised: August 2023
Board Approval Date: August, 2023

Course Description

Algebra 1 is a foundational course in mathematics that introduces students to the fundamental concepts and techniques of algebra. The course focuses on the study of linear and quadratic equations, inequalities, functions, and their graphs. In this course, students will develop the skills necessary to solve mathematical problems and communicate their findings using precise mathematical language.

The course begins with a review of operations and the properties of real numbers. Students will then learn how to solve linear equations and inequalities, and will explore the concepts of slope and intercept in linear functions. They will also learn how to graph linear equations and solve systems of linear equations using various methods, including substitution and elimination.

In the second part of the course, students will study quadratic and exponential equations, their graphs, and their applications. They will learn how to factor quadratic expressions and solve quadratic equations using various methods, including factoring, completing the square, and the quadratic formula. Students will also study functions, including domain, range, and inverse functions.

In the last part of the course, students will study trigonometry, where they will learn about the relationships between angles and sides of right triangles. They will also learn about data analysis and probability, including the use of statistical measures to analyze and interpret data, and the concept of probability and its applications.

Throughout the course, students will use algebraic techniques to solve real-world problems, including distance, rate, and time problems, and problems involving proportional relationships. They will develop problem-solving and critical thinking skills, and use technology to enhance their understanding of algebra, including graphing calculators and computer software.

By the end of the course, students will have a solid understanding of algebraic concepts and techniques, and will have developed the ability to apply them to real-world situations. They will have developed the skills to analyze, solve problems, and communicate their findings using precise mathematical language. This course is designed to prepare students for advanced studies in mathematics, science, and engineering.

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Curriculum Maps

Unit 1: Relationships between quantities and reasoning with equations

Desired Outcomes

NJSLS Mathematics

🟩 Major Content
🔷 Supporting Content
🟡 Additional Content
+ College and Career Readiness

Number and Quantity

- The Real Number System (N-RN)
  - 🟡 Use properties of rational and irrational numbers (N-RN.3).
- Quantities (N-Q)
  - 🔷 Reason quantitatively and use units to solve problems (N-Q.1, N-Q.2, N-Q.3).

Algebra

- Seeing Structure in Expressions (A-SSE)
  - 🟩 Interpret the structure of expressions (A.SSE.1, A.SSE.2).
- Creating Equations (A-CED)
  - 🟩 Create equations that describe numbers or relationships (A-CED.1, A-CED.2, A-CED.4).
- Reasoning with Equations and Inequalities (A-REI)
  - 🟩 Understand solving equations as a process of reasoning and explain the reasoning (A-REI.1).
  - 🟩 Solve equations and inequalities in one variable (A-REI.3).
  - 🟩 Represent and solve equations and inequalities graphically (A-REI.10).

Functions

- Interpreting Functions (F-IF)
  - 🟩 Understand the concept of a function and use function notation (F-IF.1, F-IF.2).
  - 🟩 Interpret functions that arise in applications in terms of the context (F-IF.4, F-IF.5).

NJSLS Mathematical Practices

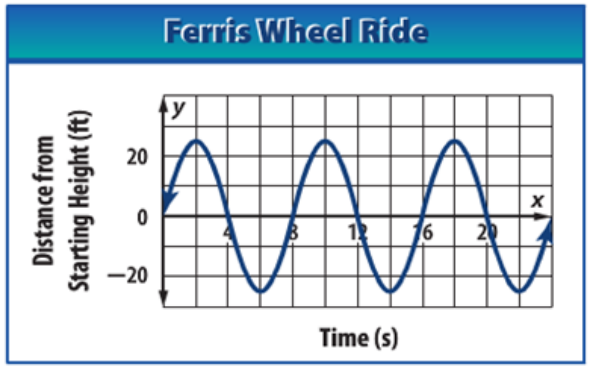
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Enduring Understandings:	**Essential Questions:**
○ Real-world phenomena can be represented efficiently in algebra by using symbols and operations. These symbols may represent unknown quantities which may or may not vary. ○ Equations and inequalities can be transformed into equivalent forms so that solutions can be found.	○ How can mathematical ideas be represented? ○ Why is it helpful to represent the same mathematical idea in different ways?
Students will know:	**Students will be able to:**
○ Algebraic expressions contain one or more numbers and variables along with arithmetic operations. They can be written as mathematical expressions or verbal expressions. They do not contain an equal sign. ○ When evaluating an expression, the set of rules	*Expressions, Equations, and Functions* ○ Write verbal expressions for algebraic expressions and vice versa. ○ Evaluate numerical and algebraic expressions using the order of operations. ○ Recognize properties of equality, identity,

that specifies which operation to do first is the order of operations. ○ Identity and equality properties can be used to justify each step when evaluating expressions and solving equations. ○ The Distributive Property can be used to evaluate numerical expressions and simplify algebraic expressions. ○ An equation is an algebraic statement that contains an equal sign. ○ A relation can be represented as a set of ordered pairs, (x, y), as an equation, table, mapping, or graph. ○ A function is a relationship between input and output in which each input value has exactly one output. ○ To interpret a graph, estimate and interpret key features. ○ Critical vocabulary is necessary to communicate and understand mathematics in the real world. ○ Variables can be used to represent an unknown amount when writing equations. ○ Solving an equation means finding the value of the variable in the equation that makes the equation true. ○ Solving equations can be done using the Properties of Equality and the strategy of undoing each operation by working backward.	Commutative, and Associative properties. ○ Use the Distributive Property to evaluate expressions and simplify expressions. ○ Solve equations with one and two variables. ○ Represent relations and interpret graphs of relations. ○ Determine whether a relation is a function and find function values. ○ Interpret intercepts, symmetry, positive, negative, increasing, decreasing, extrema, and end behavior of graphs of functions. *Linear Equations* ○ Translate sentences into equations, vice versa. ○ Solve equations by using addition, subtraction, multiplication, and division. ○ Solve equations involving more than one operation and involving consecutive integers. ○ Solve equations with the variable on each side and grouping symbols. ○ Evaluate absolute value expressions and equations. ○ Compare ratios and solve proportions. ○ Find the percent of change and solve problems involving percent of change. ○ Solve equations for given variables and use formulas to solve real-world problems. ○ Solve problems involving weighted averages.
Assessment Evidence	
<u>Suggested Performance Tasks:</u> ○ Sketch the graph of a function with one relative	**<u>Required District/State Assessments:</u>** ○ Unit Assessment

maximum and one relative minimum that could represent a real-world function. Label each axis and include appropriate units, then identify and interpret the relative extrema of your graph.

- At the beginning of a Ferris wheel ride, a passenger cart is located at the same height as the center of the wheel. The position y in feet of this cart relative to the center t seconds after the ride starts is given by the function graphed at the right. Identify and interpret the key features of the graph. Look for a pattern in the graph to help you describe its end behavior.



- NJSLA Algebra 1
- SGO Assessments

Suggested Formative/Summative Assessments:

- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/ Concept Mapping
- Presentations
- Role Playing
- Teacher-Student and Student-Student Conferencing
- Homework

Learning Plan

Learning Activities:

- NJSLA Released Items
- Starter exercises
- Guided notes

- In class activities (matching, scavenger hunt, interactive exercises, etc.)
- Variety of instructional strategies (inquiry, cooperative groups, peer editing, blended learning)
- Technology (Khan Academy, IXL, Desmos, ConnectEd, DeltaMath etc.)
- Homework relating to current topic

Related Standards

Interdisciplinary connections

Science Connections:

- Physics (HS-PS2-1): Acceleration is the measure of how fast a velocity is changing. The formula for acceleration is $a = (v_f - v_i)/t$. a represents the acceleration rate, v_f is the final velocity, v_i is the initial velocity, and t represents the time in seconds. (a) Solve the formula for the final velocity. (b) What is the final velocity of a runner who is accelerating at 2 feet per second squared for 3 seconds with an initial velocity of 4 feet per second?
- Boyle's Law (MS-PS1-4, HS-PS1-3): Students will explore rational numbers and functions in the context of Boyle's Law.
- Earth Systems (ESS3.C): Estimating Your Home's Carbon Footprint: Students graph the tabulated number of kilograms of carbon dioxide generated per kilowatt-hour, and use this graph to convert a monthly electricity bill into kilograms of carbon dioxide generated.

Physical Education Connection:

- Throwing Baseballs (2.2.12.MSC.2): Students will analyze the time, pathway and height of a baseball as the players compete.

Technology (NJSLS Computer Science and Design Thinking)

- 8.2.8.ED.2: Identify the steps in the design process that could be used to solve a problem.
  - ⇒ Example: Students can explain the significance of tables and formulas of weighted averages to a context and communicate that information in Google Sheets.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- Information And Media Literacy 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions
  - ⇒ Example: Students will apply prior knowledge when solving real world problems. Students will make sound judgments about the use of specific tools, such as Graphing Calculators, to explore and deepen understanding the concepts related to linear equations and inequalities.
- Career Awareness and Planning 9.2.12.CAP.13: Analyze how the economic, social, and political conditions of a time period can affect the labor market.
  - ⇒ Example: Students will evaluate to see how state employment has changed in the past 30 years. (Interactive Map, Worksheet)

NJ SEL Competencies

- Self-Management: Recognize the skills needed to establish and achieve personal and educational goals
- Relationship Skills: Identify who, when, where, or how to seek help for oneself or others when needed

Climate Change

- Annual Electricity Consumption in a Home (Problem #15) – Students will analyze historical electricity use consumption and calculate the tons of carbon dioxide produced by each electrical system.

Culturally Relevant Connections

- *Integrate Relevant Word Problems*: Contextualize equations using word problems that reference student interests and cultures. Example: When learning about the significance of the different elements of the equation, problems that relate to student interests such as music, sports and art enable the students to understand and relate to the concept in a more meaningful way.
- *Everyone has a Voice*: Create a classroom environment where students know that their contributions are expected and valued. Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.
- *Present New Concepts Using Student Vocabulary*: Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for

students to remember. Use multi-modal activities, analogies, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:

- Practice the thinking and procedure involved in isolating/highlighting a variable in a formula with students.
- Model the thinking process and steps involved when solving linear equations in context
- Encourage students to justify their reasoning. Provide students with sentence stems if needed.
- Model how to determine what the parts of an expression mean in context by decomposing the expression. Work toward students being able to provide this explanation with little or no prompting.
- Provide opportunities for students to practice rewriting expressions in equivalent forms. Make sure to use context based problems to facilitate understanding and retention.
- Anchor charts to model strategies and process
- Reference sheets that list formulas, step-by-step procedures and model strategies
- Conceptual word wall that contains definition, translation, pictures and/or examples
- Graphic organizers to help students interpret the meaning of terms in an expression or equation in context

ELL:

- Sentence stems to provide additional language support
- Use interactive tools such as manipulatives and technology while working in small groups to build language as well as math skills.
- The students can explain through a “think aloud” and demonstrate how they solved the problem.
- Use of word/picture walls in the classroom displaying a list of key academic vocabulary words for reference (from a specific unit).
- Use of teacher created reference sheets during task completion to check expectations, verify content and support language acquisition.
- Provide students with visuals aids like pictures and diagrams to illustrate the parts of an expression.
- Appropriate and consistent language support for the development of academic vocabulary and conceptual understanding is necessary.
- Identify key phrases or new vocabulary to pre-teach.
- Teacher models the thinking process used and the academic vocabulary needed to solve multistep problems that require students to interpret units consistently and accurately.

○ Translation dictionary ○ Teacher modeling. ○ Highlight and label solution steps for multi-step problems in different colors ○ Create an interactive notebook with students with a table of contents so they can refer to previously taught material readily ○ Algebra tiles ○ Graphing calculator ○ Videos to reinforce skills and thinking behind concepts ○ Access to tools such as tables, graphs and charts to solve problems	○ Modify the linguistic complexity of tasks by rephrasing math problems. ○ Provide support for solving linear equations and inequalities by using anchor charts with important terms, problem solving approaches, pictures and translations as needed. ○ Incorporate writing activities such as math journals to support the acquisition of academic language in mathematics and to empower students with a resource for later reference.

Enrichment

- Challenge problems from resource sets
- Extended learning goals:
  - ⇒ Students will be able to extend knowledge of interpreting graphs of functions to identifying functions as even, odd, or neither.
  - ⇒ Students will extend their knowledge of solving equations with variables on both sides to solving and identifying equations as an identity.
  - ⇒ Students will extend their knowledge of ratios and proportions to applying geometric theorems to solve for unknown angles in triangles.

Unit 2: Linear Relationships

Desired Outcomes

NJSLS Mathematics

🟩 Major Content
🔷 Supporting Content
🟡 Additional Content
+ College and Career Readiness

Number and Quantity

- Quantities (N-Q)
  - 🔷 Reason quantitatively and use units to solve problems (N.Q.1).

Algebra

- Creating Equations (A-CED)
  - 🟩 Create equations that describe numbers or relationships (A-CED.1, A-CED.2, A-CED.3).
- Reasoning with Equations and Inequalities (A-REI)
  - 🟩 Solve equations and inequalities in one variable (A-REI.3).
  - 🟡 Solve systems of equations (A-REI.5, A-REI.6).
  - 🟩 Represent and solve equations and inequalities graphically (A-REI.10, A-REI.11, A-REI.12).

Functions

- Interpreting Functions (F-IF)
  - 🟩 Understand the concept of a function and use function notation (F-IF.2, F-IF.3).
  - 🟩 Interpret functions that arise in applications in terms of the context (F-IF.4, F-IF.6).
  - 🔷 Analyze functions using different representations (F-IF.7a, F-IF.9).
- Building Functions (F-BF)
  - 🔷 Build a function that models a relationship between two quantities (F-BF.1, F-BF.2).
  - 🟡 Build new functions from existing functions (F-BF.4a).
- Linear, Quadratic, and Exponential (F-LE)

- 🔷 Construct and compare linear, quadratic, and exponential models and solve problems (F-LE.1a, F-LE.1b, F-LE.2).

Statistics and Probability

- Interpreting Categorical and Quantitative Data (S-ID)
  - 🔷 Summarize, represent, and interpret data on two categorical and quantitative variables (S-ID.6).
  - 🟩 Interpret linear models (S-ID.7, S-ID. 8, S-ID.9).

NJSLS Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Enduring Understandings:

- The symbolic language of algebra is used to communicate, analyze, and generalize patterns.
- Algebraic relationships can be represented graphically, numerically, symbolically, or verbally.
- Mathematical models can be used to describe physical relationships.
- In mathematical relationships, the value for one quantity depends on the value of another quantity.
- Ratios can show a relationship between changing quantities.

Essential Questions:

- Why is math used to model real-world situations?
- How are symbols useful in mathematics? What mathematical symbols do you know?
- How can the average rate of change be determined graphically, numerically, symbolically, or verbally?

Students will know:	**Students will be able to:**
○ Real-world situations can be graphed and interpreted using an algebraic model. ○ Linear functions can be written in a variety of forms. ○ Slope is a rate of change. ○ Real life situations can be interpreted and represented using inequalities. ○ Selecting and using the appropriate methods to solve systems of equations and inequalities demonstrates efficiency. ○ A linear system has one, infinitely many, or no solution	*Linear Functions* ○ Identify linear equations, intercepts, and zeros and graph linear equations. ○ Solve linear equations and estimate solutions to an equation by graphing. ○ Use rate of change to solve problems and find the slope of a line. ○ Write, graph, and solve problems involving direct variation. ○ Recognize arithmetic sequences and relate arithmetic sequences to linear functions. *Equations of Linear Functions* ○ Write, graph, and model real-world data with linear equations in slope-intercept form. ○ Write an equation of a line in slope-intercept form given the slope and one point and given two points. ○ Write equations of lines in point-slope form. ○ Write an equation of the line that passes through a given point, parallel and perpendicular to a given line. ○ Investigate relationships between quantities by using points on scatter plots. ○ Write equations of best-fit lines using linear regression. Use lines of fit to make and evaluate predictions. ○ Find the inverse of a relation and of a linear function. *Linear Inequalities* ○ Solve linear inequalities involving more than one operation, including using addition, subtraction, multiplication, division, and the Distributive

	Property. ○ Solve compound inequalities containing the word “and” or “or” and graph the solutions. ○ Solve and graph absolute value inequalities (> and <). ○ Graph and solve inequalities in two variables. *Systems of Linear Equations and Functions* ○ Solve systems of linear equations by graphing, substitution, and eliminations (addition, subtraction, multiplications). ○ Solve systems of linear inequalities by graphing. ○ Solve real world problems using systems of equations. ○ Determine the best method for solving systems of equations.

Assessment Evidence

Suggested Performance Tasks:	**Required District/State Assessments:**
○ Real World Linear Equation: Draw a graph representing a real-world linear function and write an equation for the graph. Describe what the graph represents. ○ Points on a Graph: a) Suppose that y=2x-3. The following points lie on the graph of this equation: A (a, 2a-3), B(b, 2b-3), and C(c, 2c-3). Amy claims that the slope of line segments AB, BC, and AC are equal. Prove that Amy’s claim is correct. Show your work and explain your reasoning. b) Are the points (-1, 1) and (1,-1) on the graph of y=2x-3? Show your work and explain your reasoning. Enter your answer, your work, and your explanation in the space provided.	○ Unit Assessment ○ NJSLA Algebra 1 ○ SGO Assessments **Suggested Formative/Summative Assessments:** ○ Describe Learning Vertically ○ Identify Key Building Blocks ○ Make Connections (between and among key building blocks) ○ Short/Extended Constructed Response Items ○ Multiple-Choice Items (where multiple answer choices may be correct) ○ Drag and Drop Items

- Quality Control Technician: A quality control technician at a candle factory tested eight 16-ounce candles, each 3 inches in diameter. These candles came from the same production run. The table shows the decrease in weight of each candle after burning for 3 hours. Candle makers believe that the rate at which the candles burn is constant. Write an equation that can be used to model the weight w of such a candle as a function of the number h of hours burning. Then, explain how the equation can be used to predict the weight of a candle that has burned for 5 hours.

Candle	1	2	3	4	5	6	7	8
Weight loss (ounces)	0.5	0.6	0.5	0.7	0.7	0.5	0.5	0.6

- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/ Concept Mapping
- Presentations
- Role Playing
- Teacher-Student and Student-Student Conferencing
- Homework

Learning Plan

Learning Activities:

- NJSLA Released Items
- Starter exercises
- Guided notes
- In class activities (matching, scavenger hunt, interactive exercises, etc.)
- Variety of instructional strategies (inquiry, cooperative groups, peer editing, blended learning)
- Technology (Khan Academy, IXL, Desmos, ConnectEd, DeltaMath etc.)
- Homework relating to current topic

Related Standards

Interdisciplinary connections

Science Connections:

- Weather (HS-ESS2-D): Use weather data and maps to utilize unit rates.
- Global Average Temperatures (HS-PS1-1;HS-PS1-2; HS-PS1-3; HS-PS1-4; HS-PS1-5; HS-PS1-7; HS-PS1-8; HS-ESS3-1): In groups (each group graphs a different decade of data), students construct graphs of global average temperatures over a decade, draw best fit lines, and compare the graphs to solar irradiance data. https://serc.carleton.edu/integrate/teaching_materials/global_energy/activity1.html

Physical Education Connection:

- Free Throws (2.2.12.MSC.2): Use basketball data to create scatter plot and lines of best fit.

Technology (NJSLS Computer Science and Design Thinking)

- 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.
  - ⇒ Example: Use digital tools to graph linear systems of inequalities that model a solution to a real-world situation. Students can then write a position statement to justify their solution, mathematical thinking, and modeling with peers.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- Critical Thinking and Problem Solving: Identify problem-solving strategies used in the development of an innovative product or practice (9.4.12.CT.1)
  - ⇒ Example: Students choose and justify a method for solving a system of equations and communicate their plan to group members.
- Career Awareness: Investigate how continuing education contributes to one's career and personal growth (9.2.12.CAP.3).
  - ⇒ Example: In this activity, students will be able to graph linear equations about real-world wage scenarios and the minimum wage.

NJ SEL Competencies

- Self-Management: Identify and apply ways to persevere or overcome barriers through alternative methods to achieve one's goals.
- Relationship Skills: Identify who, when, where, or how to seek help for oneself or others when needed.

Climate Change

- Estimating Your Home's Carbon Footprint: Students graph the tabulated number of kilograms of carbon dioxide generated per kilowatt-hour, and use this graph to convert a monthly electricity bill into kilograms of carbon dioxide generated. (ESS3.C)
- Global Average Temperatures: In groups (each group graphs a different decade of data), students construct graphs of global average temperatures over a decade, draw best fit lines, and compare the graphs to solar irradiance data. (HS-PS1-1;HS-PS1-2; HS-PS1-3; HS-PS1-4; HS-PS1-5; HS-PS1-7; HS-PS1-8; HS-ESS3-1)

Culturally Relevant Connections

- *Everyone has a Voice*: Create a classroom environment where students know that their contributions are expected and valued. Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.
- *Present New Concepts Using Student Vocabulary*: Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, visual cues, graphic representations, gestures, pictures, practice and cognates. Model to students that some vocabulary has multiple meanings. Have students create the Word Wall with their definitions and examples to foster ownership.
- *Establish Inclusion*: Highlight how the topic may relate or apply to students. Example: After a brief explanation of slope, have students come up with examples of slope at home, in their neighborhood and outside of their neighborhood. After having a volunteer list a few in each category, use the examples in class with the students. Establishing inclusion also involves regularly grouping students with different classmates to share unique perspectives.
- *Integrate Relevant Word Problems*: Contextualize equations using word problems that reference student interests and cultures. Example: When learning different types of functions, problems that relate to student interests such as music, sports and art enable the students to understand and relate to the concept in a more meaningful way.
- *Encourage Student Leadership*: Create an avenue for students to propose problem solving strategies and potential projects. Example: Students can learn about different function types by creating problems together

and deciding if the problems fit the necessary criteria. This experience will allow students to discuss and explore their current level of understanding by applying the concepts to relevant real-life experiences.

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:

- Represent linear models in multiple forms: tables, graphs, arithmetic sequences, and equations. Model how to convert between forms.
- Demonstrate how to determine a rate of change, in context. Work towards students being able to find and describe, using units, and rates of change with little or no prompting.
- Model how to determine the parts of a linear equation in context and how to identify the correct form of the equation. Work towards students being able to convert between forms with little or no prompting.
- Give students opportunities to explore and discover properties of systems of linear models, i.e. lines with the same slope, lines with different slopes, and lines with negative reciprocal slopes, in multiple forms.
- Have students collect real world data and determine a linear model to fit the data. Justify the validity of the linear model. Work towards students being able to determine the strength of a linear model and interpolate with little or no prompting.
- Model the thinking process and steps involved when solving linear inequalities in context

ELL:

- Sentence stems to provide additional language support
- Use interactive tools such as manipulatives and technology while working in small groups to build language as well as math skills.
- The students can explain through a “think aloud” and demonstrate how they solved the problem.
- Use of word/picture walls in the classroom displaying a list of key academic vocabulary words for reference (from a specific unit).
- Use of teacher-created reference sheets during task completion to check expectations, verify content and support language acquisition.
- Appropriate and consistent language support for the development of academic vocabulary and conceptual understanding is necessary.
- Identify key phrases or new vocabulary to pre-teach.
- Teacher models the thinking process used and the academic vocabulary needed to solve multistep problems that require students to interpret units consistently and accurately.
- Modify the linguistic complexity of tasks by rephrasing math problems.
- Provide support for graphing linear equations and inequalities, converting between linear forms,

○ Represent the constraints of a linear inequality, in context. Model how to determine these constraints on the graph of the inequality. ○ Model how to solve one system of linear equations in multiple ways and give students opportunities to choose a method for solving systems. ○ Encourage students to justify their reasoning. Provide students with sentence stems if needed. ○ Anchor charts to model strategies and process ○ Reference sheets that list formulas, step-by-step procedures and model strategies ○ Conceptual word wall that contains definition, translation, pictures and/or examples ○ Graphic organizers to help students interpret the meaning of terms in an expression or equation in context ○ Translation dictionary ○ Teacher modeling. ○ Highlight and label solution steps for multi-step problems in different colors ○ Create an interactive notebook with students with a table of contents so they can refer to previously taught material readily ○ Algebra tiles ○ Graphing calculator ○ Videos to reinforce skills and thinking behind concepts ○ Access to tools such as tables, graphs and charts to solve problems	determining a line of best fit, and solving systems of equations and inequalities by using anchor charts with important terms, problem solving approaches, pictures and translations as needed. ○ Incorporate writing activities such as math journals to support the acquisition of academic language in mathematics and to empower students with a resource for later reference. ○ Prompt students to use an organizational chart to answer multi-step or multi-part word problems.
Enrichment	
○ Challenge problems from resource sets ○ Extended learning goals:	

- ⇒ Explore transformations of linear equations in the coordinate plane, recognizing when a line is translated up, down, left, and right.
- ⇒ Determine and apply a formula to find the sum of any arithmetic sequence.
- ⇒ Determine the tangent line of a curve.
- ⇒ Determine when a function is one-to-one or onto.
- ⇒ Apply absolute value to precision work.
- ⇒ Apply linear programming for systems of linear inequalities to minimize and maximize.
- ⇒ Determine the intersection of two parabolas using substitution.
- ⇒ Solve a system of equations in 3 variables.
- ⇒ Use a matrix and Cramer's Rule to determine the solution to a system of equations.
- ⇒ Describe the area of a polygon using a system of linear inequalities.

Unit 3: Exponential and Quadratic Functions

Desired Outcomes

NJSLS Mathematics

🟩 Major Content
🔷 Supporting Content
🟡 Additional Content
+ College and Career Readiness

Number and Quantity
- The Real Number System (N-RN)
    - 🟡 Extend the properties of exponents to rational exponents (N-RN.1, N-RN.2).

Algebra
- Seeing Structure in Expressions (A-SSE)
    - 🟩 Interpret the structure of expression (A-SSE.1a, A-SSE.2).
    - 🔷 Write expressions in equivalent forms to solve problems (A-SSE.3a, A-SSE.3b).
- Arithmetic with Polynomials and Rational Expressions (A-APR)
    - 🟩 Perform arithmetic operations on polynomials (A-APR.1).
- Reasoning with Equations and Inequalities (A-REI)
    - 🟩 Understand solving equations as a process of reasoning and explain the reasoning (A-REI.1).
    - 🟩 Solve equations in one variable (A-REI.4a).

Functions
- Interpreting Functions (F-IF)
    - 🟩 Interpret functions that arise in applications in terms of the context (F-IF.4, F-IF.6).
    - 🟩 Understand the concept of a function and use function notation (F-IF.3).
    - 🔷 Analyze functions using different representations (F-IF.7a, F-IF.7e, F-IF.8b).
    - 🔷 Analyze functions using different representations (F-IF.7b, F-IF.8a).
- Building Functions (F-BF)

        - 🟡 Build a function that models a relationship between two quantities (F-BF.2).
    - Linear, Quadratic, and Exponential (F-LE)
        - 🔷 Construct and compare linear, quadratic, and exponential models and solve problems (F-LE.1a, F-LE.1b, F-LE.1c, F-LE.2)

NJSLS Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Enduring Understandings:	**Essential Questions:**
○ non-linear change can be represented by graphs, mathematical models, and equations. ○ an exponential function can model growth or decay of an initial amount. ○ in mathematical relationships, the value for one quantity depends on the value of another quantity. ○ a relationship exists between the laws of exponents and scientific notation.	○ How can mathematical language be used to describe non-linear change? Why do we use different methods to solve math problems? ○ How can we model situations using exponents? ○ When could a nonlinear function be used to model a real-world situation? ○ Why are graphs useful? ○ How are the rules of exponents used when multiplying and dividing expressions that are written in scientific notation?
Students will know:	**Students will be able to:**
○ an exponential function can model growth or decay in real life when describing situations such as population growth, compound interest, and car	*Exponents and Exponential Functions* ○ simplify and perform operations on expressions involving exponents (product of powers, power of

depreciation.
- the graph of an exponential function of the form $y=ab^x$ will never equal zero and will have exactly one y -intercept.
- in the exponential function $y=ab^x$, a represents the initial amount and b represents the growth/decay factor.
- compound interest can be found using the formula $A=P(1+r/n)^{nt}$, where A represents the balance, P represents the principal (initial deposit), r represents the annual interest rate (expressed as a decimal), n represents the number of times interest is compounded yearly, and t represents the time in years.
- quadratic functions can be graphed using ordered pairs, graphed using the roots and vertex, written and graphed from different forms using algebraic and calculator techniques, transformed and reflected both graphically and algebraically, related to real-world situations by graphing and interpreting algebraic models, used to analyze data, used to determine whether it represents a function and the type of function.
- solving a quadratic function will produce the roots of the parabola.
- the ratio (-b/2a) is the x-coordinate at the vertex.

power, power of product, quotient of powers, power of a quotient, zero exponent property, negative exponent property).
- extend the properties of integer exponents to rational exponents to evaluate and rewrite expressions and solve equations.
- express, multiply/divide, and apply numbers in scientific notation.
- graph and use exponential functions in growth and decay applications.
- identify and generate geometric sequences.
- use and write recursive formulas for arithmetic and geometric sequences

Quadratic Expressions, Equations, and Functions
- write polynomials in standard form.
- add, subtract, and multiply (Distributive Property, squares of sums and differences, product of a sum and a difference) polynomials.
- solve equations involving the products of monomials and polynomials.
- factor trinomial expressions and solve quadratic equations for real solutions by factoring (Distributive Property, ax2+bx, x2+bx+c, ax2+bx+c, difference of squares, perfect square trinomials, perfect squares).
- graph quadratic functions.
- apply translations, dilations, and reflections to quadratic functions.
- solve quadratic equations for real solutions by graphing, completing the square, and using the Quadratic Formula.
- use the discriminant of the Quadratic Formula to determine the number of real solutions to a quadratic equation.

	○ identify linear, quadratic, and exponential functions from given data and write equations that model data. ○ identify and graph special functions (step functions, absolute value functions, piecewise-defined functions).

Assessment Evidence

Suggested Performance Tasks:

- Techniques for Solving a Quadratic Equation: Compare and contrast the following strategies for solving x^2 -5x - 7 = 0: completing the square, graphing, and factoring.
- Correct Model: Two websites launched on the same day. At the end of the first week, the number of visitors to each website was 25. For the first eight weeks, the number of visitors to each website increased according to the corresponding rules. Website A: The number of visitors doubled each week. Website B: The number of visitors increased by 150 each week.
  - ⇒ Part A: Make a table to show the number of visitors to each website for the first eight weeks.
  - ⇒ Part B: Based on the data for the first eight weeks, Jose claims that the number of visitors to each website can be modeled as a linear function of the number of weeks online. For each website, decide if Jose's claim is correct. If it is correct, explain why. If it is not correct, explain why and describe a more appropriate model.

Required District/State Assessments:

- Unit Assessment
- NJSLA Algebra 1
- SGO Assessments

Suggested Formative/Summative Assessments:

- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/ Concept Mapping
- Presentations

	○ Role Playing ○ Teacher-Student and Student-Student Conferencing ○ Homework

Learning Plan

Learning Activities:

- NJSLA Released Items
- Starter exercises
- Guided notes
- In class activities (matching, scavenger hunt, interactive exercises, etc.)
- Variety of instructional strategies (inquiry, cooperative groups, peer editing, blended learning)
- Technology (Khan Academy, IXL, Desmos, ConnectEd, DeltaMath etc.)
- Homework relating to current topic

Related Standards

Interdisciplinary connections

Social Studies Connections:

- Population and Food Supply (6.2.12.GeoPP.6.a): Students will analyze the population and food production growth and analyze the main challenge of food shortages.

Financial Literacy Connection:

- Stock Prices (9.1.12.PB.1): Students will compare the performance of different stocks to compare investment decisions with savings decisions.

Technology (NJSLS Computer Science and Design Thinking)

- 8.1.8.DA.1: Organize and transform data collected using computational tools to make it usable for a specific purpose.
  - ⇒ Example: Modeling the Keeling Curve with Excel (Problem #22): Students will use spreadsheets to

analyze historical atmospheric carbon dioxide measurements.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.2.12.CAP.14: Analyze and critique various sources of income and available resources (e.g., financial assets, property, and transfer payments) and how they may substitute for earned income.
  - ⇒ Example: Students will apply their knowledge of exponential functions to analyze return on investment and inflation/depreciation.
- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
  - ⇒ Example: Students will apply prior knowledge when solving real world problems. Students will make sound judgments about the use of specific tools, such as Graphing Calculators, to explore and deepen understanding the concepts related to quadratic equations and functions including how to solve, write, graph, interpret and explain these relationships accurately.

NJ SEL Competencies

- Self-Management: Recognize the impact of one's feelings and thoughts on one's own behavior.
- Social Awareness: Demonstrate an understanding of the need for mutual respect when viewpoints differ.

Climate Change

- Modeling the Keeling Curve with Excel (Problem #22): Students will use spreadsheets to analyze historical atmospheric carbon dioxide measurements.

Culturally Relevant Connections

- *Integrate Relevant Word Problems*: Contextualize equations using word problems that reference student interests and cultures. Example: When learning about the significance of the different elements of the equation, problems that relate to student interests such as music, sports and art enable the students to understand and relate to the concept in a more meaningful way.
- *Everyone has a Voice*: Create a classroom environment where students know that their contributions are expected and valued. Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the

classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.

- *Present New Concepts Using Student Vocabulary*: Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:

- Encourage students to justify their reasoning. Have students come up with sentence stems (i.e. "I already know / I think this is similar to).
- Have students determine the characteristics of exponential and quadratic functions. Work towards students being able to recognize these functions with little or no prompting.
- Use real-life examples to enhance understanding and retention.
- Use anchor charts to encourage independent learning
- Provide reference sheets that list formulas, step-by-step procedures, and model strategies
- Display a conceptual word wall that contains definition, translation, pictures, and/or examples
- Provide graphic organizers to help students interpret the meaning of terms
- Use a translation dictionary
- Teacher modeling
- Create an interactive notebook with a table of

ELL:

- Use sentence stems to provide additional language support.
- Use interactive tools such as manipulatives and technology while working in small groups to build language as well as math skills.
- Allow students opportunities to explain through a "think aloud" and demonstrate how they solved the problem.
- Use of word/picture walls in the classroom displaying a list of key academic vocabulary words for reference (from a specific unit).
- Use teacher-created reference sheets during task completion to check expectations, verify content, and support language acquisition.
- Provide students with visuals aids like pictures and diagrams
- Appropriate and consistent language support for the development of academic vocabulary and conceptual understanding is necessary.
- Identify key phrases or new vocabulary to pre-teach.

contents so they can refer to previously taught material readily ○ Varied use of graphing calculator ○ Use of videos to reinforce skills and thinking behind concepts ○ Provide access to tools such as tables, graphs, and charts to solve problems ○ Use algebra tiles to model operations on quadratic polynomials	○ When applying academic vocabulary, the teacher models the thinking process used to solve multistep problems that require students to interpret units consistently and accurately. ○ Modify the linguistic complexity of tasks by rephrasing math problems. ○ Incorporate writing activities such as math journals to support the acquisition of academic language in mathematics and to empower students with a resource for later reference.

Enrichment

- Challenge problems from resource sets
- Extended learning goals:
  - ⇒ Students will be able to extend knowledge of scientific notation with engineering notation (see 7.4 Enrichment).
  - ⇒ Students can compare and contrast exponential functions with logarithmic functions to see how they are inverses of each other (see 7.5 Enrichment).

Unit 4: Data Analysis

Desired Outcomes

Established Goals: NJSLS-Mathematics

🟩 Major Content
🔷 Supporting Content
🟡 Additional Content
+ College and Career Readiness

Statistics and Probability

- Interpreting Categorical and Quantitative Data (S-ID)
  - 🟡 Summarize, represent, and interpret data on a single count or measurement variable (S-ID.1, S-ID.2, S-ID.3).
  - 🔷 Summarize, represent, and interpret data on two categorical and quantitative variables (S-ID.5).
- Making Inferences and Justifying Conclusions (S-IC)
  - 🟡 Understand and evaluate random processes underlying statistical experiments (S-IC.2).
  - 🟡 Make inferences and justify conclusions from sample surveys, experiments, and observational studies (S-IC.3, S-IC.5, S-IC.6).
- Conditional Probability and the Rules of Probability (S-CP)
  - 🟡 Understand independence and conditional probability and use them to interpret data (S-CP.1, S-CP.2, S-CP.3, S-CP.4, S-CP.5).
  - + Use the rules of probability to compute probabilities of compound events (S-CP.6, S-CP.7, S-CP.8, S-CP.9).
- Using Probability to Make Decisions (S-MD)
  - + Use probability to evaluate outcomes of decisions (S-MD.5).
  - + Use probability to evaluate outcomes of decisions (S-MD.6).

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Enduring Understandings:	**Essential Questions:**
○ data can be represented in different forms and the best representation of the data will depend on the type of data ○ the way real-world data is collected may yield misleading results.	○ How are statistics and probability used in the real world? ○ How can you make good decisions? What factors can affect good decision making? ○ How does the way data is analyzed or communicated influence the way it is interpreted? ○ How can different measures be used to interpret and compare sets of data?
Students will know:	**Students will be able to:**
○ Key vocabulary: sample, population, simple random, systematic, self-selected, convenience, stratified sample ○ A statistic is a measure that describes a characteristic of a sample. ○ A permutation is an arrangement or listing in which the order or placement of the arrangement is important. ○ A combination is an arrangement or listing in which the order or placement is not important. ○ How to construct a histogram and a box-and-whiskers plot using graphing calculator	○ Classify and analyze samples and studies. ○ Identify sample statistics and population parameters. ○ Analyze data using statistics. ○ Describe and use the shape of a distribution to select appropriate statistics. ○ Determine the effect that transformations of data have on measures of central tendency and variation. ○ Compare data using measures of central tendency and variation. ○ Calculate experimental probabilities.

technology. ○ How to use different measures to interpret and compare sets of data. ○ How to analyze the composition and results of a statistical survey.	○ Design simulations and summarize data from simulations ○ Design surveys and evaluate results. ○ Use permutations and combinations. ○ Find probabilities of compound events (independent and dependent, mutually exclusive events). ○ Find the expected value of a probability distribution.
Assessment Evidence	
<u>Suggested Performance Tasks:</u> ○ Business Owner: Write a real-world story in which you are the owner of a business. Explain how you could use a probability distribution to help you make a business decision.	**<u>Required District/State Assessments:</u>** ○ Unit Assessment ○ NJSLA Algebra 1 ○ SGO Assessments **<u>Suggested Formative/Summative Assessments:</u>** ○ Describe Learning Vertically ○ Identify Key Building Blocks ○ Make Connections (between and among key building blocks) ○ Short/Extended Constructed Response Items ○ Multiple-Choice Items (where multiple answer choices may be correct) ○ Drag and Drop Items ○ Use of Equation Editor ○ Quizzes ○ Journal Entries/Reflections/Quick-Writes ○ Accountable talk ○ Projects ○ Portfolio

	○ Observation ○ Graphic Organizers/ Concept Mapping ○ Presentations ○ Role Playing ○ Teacher-Student and Student-Student Conferencing ○ Homework

Learning Plan

Learning Activities:

- NJSLA Released Items
- Starter exercises
- Guided notes
- In class activities (matching, scavenger hunt, interactive exercises, etc.)
- Variety of instructional strategies (inquiry, cooperative groups, peer editing, blended learning)
- Technology (Khan Academy, IXL, Desmos, ConnectEd, DeltaMath etc.)
- Homework relating to current topic

Related Standards

Interdisciplinary connections

Language Arts Connections (W.9-10.1.A):

- Compare and contrast the benefits of displaying data using histograms and box-and-whisker plots.
- Explain why the mean and standard deviation are used to compare the center and spread of two symmetrical distributions and the five-number summary is used to compare the center and spread of two skewed distributions or a symmetric distribution and a skewed distribution.

Technology (NJSLS Computer Science and Design Thinking)

- 8.2.8.ED.3: Develop a proposal for a solution to a real-world problem that includes a model (e.g., physical

prototype, graphical/technical sketch).
    - ⇒ Example: Students will utilize Desmos, Google Sheets, and TI-84 Graphing Calculators to represent data and relationships.

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
    - ⇒ Example: College expenses range from tuition to housing to school supplies. In this activity, students will take a look at how much these college expenses can add up and calculate the measures of central tendency for college costs.

NJ SEL Competencies

- Responsible Decision-Making: Evaluate personal, ethical, safety and civic impact of decisions
- Self-Awareness: Recognize the importance of self-confidence in handling daily tasks and challenges

Climate Change

- Carbon Dioxide Production at Home (Problem #16): In this problem set, learners will consider the "Carbon Footprint" of a family of four in a given context, as well as the US and global averages, and compare that with their own to answer a series of questions. They will make calculations from a pie graph and use an online Carbon Footprint calculator to determine their own per-capita carbon production.

Culturally Relevant Connections

- *Integrate Relevant Word Problems*: Contextualize data distribution using word problems that reference student interests and cultures. Example: When learning about standard deviation, students can survey students about their favorite movie and its length. Students can then determine and explain what the standard deviation of the movie lengths can tell you about popular movies
- *Everyone has a Voice*: Create a classroom environment where students know that their contributions are expected and valued. Example: Norms for sharing are established that communicate a growth mindset for

mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.

- *Present New Concepts Using Student Vocabulary*: Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:

- Practice the thinking and procedure of forming histograms and box-and-whisker plots. Prompt students to identify what the different aspects in each display of data represent.
- Model the thinking process and steps involved when finding the mean absolute deviation and the standard deviation.
- Encourage students to justify their reasoning. Provide students with sentence stems if needed.
- Model various probability simulations for students by using manipulative resources. (i.e. when learning about experimental vs. theoretical probability, have students create their own simulation of coin tosses to show experimental probability may not always be 50%)
- Allow students to create their own probability situations. Have students relate the probability situations they create to real-life examples.
- Anchor charts to model strategies and process

ELL:

- Sentence stems to provide additional language support
- Use interactive tools such as manipulatives and technology while working in small groups to build language as well as math skills.
- The students can explain through a “think aloud” and demonstrate how they solved the problem.
- Use of word/picture walls in the classroom displaying a list of key academic vocabulary words for reference (from a specific unit).
- Use of teacher created reference sheets during task completion to check expectations, verify content and support language acquisition.
- Provide students with visuals aids like pictures and diagrams to illustrate the similarities and differences between mean, median, mode, mean absolute deviation, and standard deviation
- Appropriate and consistent language support for the development of academic vocabulary and conceptual understanding is necessary.

- Reference sheets that list formulas, step-by-step procedures and model strategies
- Conceptual word wall that contains definition, translation, pictures and/or examples
- Graphic organizers to help students interpret the meaning of terms in an expression or equation in context
- Teacher modeling.
- Highlight and label solution steps for multi-step problems in different colors
- Create an interactive notebook with students with a table of contents so they can refer to previously taught material readily
- Graphing calculator
- Videos to reinforce skills and thinking behind concepts
- Access to tools such as tables, graphs and charts to solve problems
- Provide a wordbank for new vocabulary terms

- Identify key phrases or new vocabulary to pre-teach.
- Teacher models the thinking process used and the academic vocabulary needed to solve multistep problems that require students to interpret units consistently and accurately.
- Modify the linguistic complexity of tasks by rephrasing math problems.
- Provide support for solving linear equations and inequalities by using anchor charts with important terms, problem solving approaches, pictures and translations as needed.
- Incorporate writing activities such as math journals to support the acquisition of academic language in mathematics and to empower students with a resource for later reference.
- Prompt students to use an organizational chart to answer multi-step or multi-part word problems.
- Have visuals of any words that may have a separate meaning outside of the math classroom (i.e. the words “mean” and “table” which are commonly used in Unit 5)

Enrichment

- Challenge problems from resource sets
- Extended learning goals:
  - ⇒ Students will be able to extend their knowledge of bias by learning that it could be accidental (ex. A penny is more likely to land on heads than tails due to it being beveled or grooved slightly.
  - ⇒ Students can extend their learning on standard deviation by discovering Chebyshev’s Theorem, which identifies the two data points in which most of the data (at least 75%) in a sample size will lie
  - ⇒ Students can extend their knowledge on probability distributions by learning about binomial distributions. Example: out of 5 draws from a bag of marbles, what is the probability of pulling exactly 2 marbles

Unit 5: Advanced Functions and Equations

Desired Outcomes

NJSLS Mathematics

🟩 Major Content
🔷 Supporting Content
🟡 Additional Content
+ College and Career Readiness

Number and Quantity
- The Real Number System (N-RN)
    - 🟡 Extend the properties of exponents to rational exponents (N-RN.2).

Algebra
- Creating Equations (A-CED)
    - 🟩 Create equations that describe numbers or relationships (A-CED.2).
- Reasoning with Equations and Inequalities (A-REI)
    - 🟩 Solve equations and inequalities in one variable (A-REI.4a).

Functions
- Interpreting Functions (F-IF)
    - 🟩 Interpret functions that arise in applications in terms of context (F-IF.4).
    - 🔷 Analyze functions using different representations (F-IF.7b).

Geometry
- Similarity, Right Triangles, and Trigonometry (G-SRT)
    - \+ Define trigonometric ratios and solve problems involving right triangles (G-SRT.8).

NJSLS Mathematical Practices
1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Enduring Understandings:	**Essential Questions:**
○ radical expressions can be simplified. ○ radical equations can be solved by manipulation. ○ a simplified version of an expression may be more useful.	○ How can you choose a model to represent a real-world situation? ○ How can simplifying mathematical expressions be useful?
Students will know:	**Students will be able to:**
○ The graph of $y=a\sqrt{x}$ starts at the origin and passes through the point (1, a). ○ When the radicand contains no perfect square factors other than 1, it is said to be in simplest form. ○ Adding and subtracting expressions with radicals uses the process of combining like terms. For terms to be combined, their radicands must be the same. ○ Multiplying two radical expressions with two terms is similar to multiplying binomials. The radicands do not have to be like radicands when multiplying. ○ To solve radical equations, the radical must first be isolated on one side of the equation.	*Radical Functions* ○ graph and transform (dilations, reflections, translations) radical functions. ○ simplify (Product Property, Quotient Property), add, subtract, and multiply radical expressions with indices of 2. ○ solve radical equations (including those with extraneous solutions). *Geometry* ○ solve problems by using the Pythagorean Theorem. ○ determine whether a triangle is a right triangle. ○ find trigonometric ratios of angles. ○ use trigonometry to solve right triangles. *Rational Functions and Equations*

○ The Pythagorean Theorem is only applicable to right triangles. ○ Trigonometry is the study of relationships among the angles and sides of a right triangle. ○ A rational function is a function of the form y=p/q, where the numerator, p, and denominator, q, are both polynomials. ○ Any value that makes the value of q equal to 0 is called an excluded value of the rational function and must be excluded from the domain of the function. ○ If both sides of a rational equation are single fractions, cross products or multiplying each side of the equation by the LCD can be used to solve the resulting equation.	○ identify and use inverse variations. ○ graph inverse variations. ○ identify excluded values of rational functions. ○ identify and use asymptotes to graph rational functions. ○ solve rational equations. ○ use rational equations to solve problems.

Assessment Evidence

Suggested Performance Tasks:

- Is the following equation sometimes, always or never true? Explain. √((x-2)^2)=x-2
  Fire Department: When fighting a fire, the velocity v of water being pumped into the air is modeled by the function y=√2hg, where h represents the maximum height of the water and g represents the acceleration due to gravity (32 ft/s^2).
  - ⇒ Solve the function for h.
  - ⇒ The Medford Fire Department needs a pump that will propel water 80 feet into the air. Will a pump advertised to project water with a velocity of 70 feet per second meet their needs? Explain.

Required District/State Assessments:

- Unit Assessment
- NJSLA Algebra 1
- SGO Assessments

Suggested Formative/Summative Assessments:

- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items

⇒ The Willingboro Fire Department must purchase a pump that will propel water 90 feet into the air. Will a pump that is advertised to project water with a velocity of 77 feet per second meet the fire department's needs? Explain.	○ Use of Equation Editor ○ Quizzes ○ Journal Entries/Reflections/Quick-Writes ○ Accountable talk ○ Projects ○ Portfolio ○ Observation ○ Graphic Organizers/ Concept Mapping ○ Presentations ○ Role Playing ○ Teacher-Student and Student-Student Conferencing ○ Homework

Learning Plan

Learning Activities:

- NJSLA Released Items
- Starter exercises
- Guided notes
- In class activities (matching, scavenger hunt, interactive exercises, etc.)
- Variety of instructional strategies (inquiry, cooperative groups, peer editing, blended learning)
- Technology (Khan Academy, IXL, Desmos, ConnectEd, DeltaMath etc.)
- Homework relating to current topic

Related Standards

Interdisciplinary connections

Science Connections (HS-ESS3-1):

- Apply exponential functions to calculate the dimensions of a water rain barrel based on the velocity of water flow.

Language Arts Connection (WHST.9-12.2):

- Describe step by step how to multiply two radical expressions, each with two terms. Write an example to demonstrate your description.

Technology (NJSLS Computer Science and Design Thinking)

- 8.2.12.ED.6: Analyze the effects of changing resources when designing a specific product or system (e.g., materials, energy, tools, capital, labor).
  - ⇒ Example: Students will compare and contrast tools such as Desmos, Google Sheets, and TI-84 Graphing Calculators to create tables and graphs for radical functions..

21st Century Skills (NJSLS Career Readiness, Life Literacies, and Key Skills)

- 9.4.8.IML.3: Create a digital visualization that effectively communicates a data set using formatting techniques such as form, position, size, color, movement, and spatial grouping.
  - ⇒ Example: Students can work collaboratively to explore how transformations can be visualized both algebraically and graphically using graphing calculators. Students share their findings with classmates using Google Docs and/or Google Slides.
- 9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.
  - ⇒ Example: Students can work collaboratively on problems which involve using trigonometric ratios to find missing lengths and/or angles. In many problems that can be solved using trigonometry, there are many ways to solve said problem. Students could work collaboratively to solve the problem and identify all relevant methods that could be used.

NJ SEL Competencies

- Self-Management: Understand and practice strategies for managing one's own emotions, thoughts and behaviors
- Self-Awareness: Recognize the importance of self-confidence in handling daily tasks and challenges

Climate Change

- Paris in a Different Light (Problem #32): Students will analyze an image of Paris from the Advanced Spaceborne Thermal Emission and Reflection Radiometer (ASTER) for vegetation and buildings. Applying

trigonometry, they will determine different measurements in the image.

Culturally Relevant Connections

- *Everyone has a Voice*: Create a classroom environment where students know that their contributions are expected and valued. Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.
- *Present New Concepts Using Student Vocabulary*: Use student diction to capture attention and build understanding before using academic terms. Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.
- *History of Mathematics*: Teach the students the history and origin of the vocabulary for trigonometry. For example, the origins of the word hypotenuse comes from ancient Greece. Students could research other historical aspects of trigonometry and teachers can post the findings around the classroom.

Accommodations

Special Education/ 504/ At Risk Students Accommodations & Modifications:

- Encourage students to justify their reasoning. Have students come up with sentence stems (i.e. “I already know / I think this is similar to).
- Have students determine the characteristics of radical and rational functions. Work towards students being able to recognize these functions with little or no prompting.
- Use real-life examples to enhance understanding and retention.

ELL:

- Sentence stems to provide additional language support
- Use interactive tools such as manipulatives and technology while working in small groups to build language as well as math skills.
- The students can explain through a “think aloud” and demonstrate how they solved the problem.
- Use of word/picture walls in the classroom displaying a list of key academic vocabulary words for reference (from a specific unit).

- Use anchor charts to encourage independent learning
- Provide reference sheets that list formulas, step-by-step procedures, and model strategies
- Display a conceptual word wall that contains definition, translation, pictures and/or examples
- Provide graphic organizers to help students interpret the meaning of terms
- Utilize translation dictionary
- Teacher modeling
- Create an interactive notebook with students with a table of contents so they can refer to previously taught material readily
- Varied use of graphing calculator
- Incorporate videos to reinforce skills and thinking behind concepts
- Provide access to tools such as tables, graphs and charts to solve problems

- Use teacher-created reference sheets during task completion to check expectations, verify content and support language acquisition.
- Provide students with visuals aids like pictures and diagrams to illustrate the parts of an expression.
- Appropriate and consistent language support for the development of academic vocabulary and conceptual understanding is necessary.
- Identify key phrases or new vocabulary to pre-teach.
- Applying academic vocabulary, teacher models the thinking process used to solve multistep problems that require students to interpret rational and radical functions consistently and accurately.
- Modify the linguistic complexity of tasks by rephrasing math problems.
- Provide support for solving linear equations and inequalities by using anchor charts with important terms, problem solving approaches, pictures and translations as needed.
- Incorporate writing activities such as math journals to support the acquisition of academic language in mathematics and to empower students with a resource for later reference.

Enrichment

- Challenge problems from resource sets
- Extended learning goals:
  - ⇒ Students can extend their knowledge by learning how to graph cube root functions, identifying the similarities and differences to square root functions
  - ⇒ If students have mastered the first three trigonometric ratios (sine, cosine, and tangent), then they can

learn the other three trigonometric ratios (secant, cosecant, and cotangent). Students should also be able to relate the first group of three ratios to the second group.

⇒ Students can also extend their learning of graphs of rational functions by learning how to graph rational inequalities on the coordinate plane. Students should be able to use their prior knowledge of graphing inequalities to graph the rational inequalities.

Appendix A: Culturally Relevant Pedagogy Examples

BUILDING EQUITY IN YOUR TEACHING PRACTICE

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

CONTENT INTEGRATION	KNOWLEDGE CONSTRUCTION	PREJUDICE REDUCTION	EQUITABLE PEDAGOGY	EMPOWERING SCHOOL CULTURE
Teachers use examples and content from a variety of cultures & groups.	Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives & biases.	Teachers implement lessons and activities to assert positive images of ethnic groups & improve intergroup relations.	Teachers modify techniques and methods to facilitate the academic achievement of students from diverse backgrounds.	Using the other four dimensions to create a safe and healthy educational environment for all.
This unit / lesson is connected to other topics explored with students. There are multiple viewpoints reflected in the content of this unit / lesson. The materials and resources are reflective of the diverse identities and experiences of students. The content affirms students, as well as exposes them to experiences other than their own.	This unit / lesson provides context to the history of privilege and oppression. This unit / lesson addresses power relationships. This unit / lesson help students to develop research and critical thinking skills. This curriculum creates windows and mirrors* for students.	This unit / lesson help students question and unpack biases & stereotypes. This unit / lesson help students examine, research and question information and sources. The curriculum encourage discussion and understanding about the groups of people being represented. This unit / lesson challenges dominant perspectives.	The instruction has been modified to meet the needs of each student. Students feel respected and their cultural identities are valued. Additional supports have been provided for students to become successful and independent learners. Opportunities are provided for student to reflect on their learning and provide feedback.	There are opportunities for students to connect with the community. My classroom is welcoming and supportive for all students? I am aware of and sensitive to the needs of my students and their families. There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom.

Developed by Karla E. Vigil. Adapted with permission from James A. Banks, CULTURAL DIVERSITY AND EDUCATION: FOUNDATIONS, CURRICULUM, AND TEACHING (6th edition), New York: Routledge, 2016, page 5 and Gordon School Institute on Multicultural Practice.

Appendix B: English Language Learners

WIDA Levels:

At the given level of English language proficiency, English language learners will process, understand, produce or use

6- Reaching	• Specialized or technical language reflective of the content areas at grade level • A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level • Oral or written communication in English comparable to proficient English peers
5- Bridging	• Specialized or technical language of the content areas • A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports • Oral or written language approaching comparability to that of proficient English peers when presented with grade level material.
4- Expanding	• Specific and some technical language of the content areas • A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs • Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support
3- Developing	• General and some specific language of the content areas • Expanded sentences in oral interaction or written paragraphs • Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support
2- Beginning	• General language related to the content area • Phrases or short sentences • Oral or written language with phonological, syntactic, or semantic errors that often impede of the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support
1- Entering	• Pictorial or graphic representation of the language of the content areas • Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support

Language Development Supports For English Language Learners To Increase Comprehension and Communication Skills

Environment

- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students' background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners
- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaning and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students' lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

Sensory Supports*	Graphic Supports*	Interactive Supports*	Verbal and Textual Supports
• Real-life objects (realia) or concrete objects • Physical models • Manipulatives • Pictures & photographs • Visual representations or models such as diagrams or drawings • Videos & films • Newspapers or magazines • Gestures • Physical movements • Music & songs	• Graphs • Charts • Timelines • Number lines • Graphic organizers • Graphing paper	• In a whole group • In a small group • With a partner such as *Turn-and-Talk* • In pairs as a group (first, two pairs work independently, then they form a group of four) • In triads • Cooperative learning structures such as *Think-Pair-Share* • Interactive websites or software • With a mentor or coach	• Labeling • Students' native language • Modeling • Repetitions • Paraphrasing • Summarizing • Guiding questions • Clarifying questions • Probing questions • Leveled questions such as *What? When? Where? How? Why?* • Questioning prompts & cues • Word Banks • Sentence starters • Sentence frames • Discussion frames • Talk moves, including *Wait Time*

*from *Understanding the WIDA English Language Proficiency Standards. A Resource Guide*. 2007 Edition.. Board of Regents of the University of Wisconsin System, on behalf of the WIDA Consortium—www.wida.us.

Galina (Halla) Jmourko, ESOL Coach, PGCPS; 2015, Rvsd. 2016

Appendix C: WIDA ELD Standards Learners

ELD-MA.9-12 Explain Interpretive	Interpret mathematical explanations by • Identifying concept or entity • Analyzing data and owning problem-solving approaches • Evaluating rationales, models, and/or interpretations based on evidence and mathematical principles
ELD-MA 9-12 Explain Expressive	Construct mathematical explanations that • Introduce mathematical concept or entity • Share solutions with others • Describe data and/or approach used to solve a problem • State reasoning used to generate own or alternate solutions
ELD-MA.9-12 Argue Interpretive	Interpret mathematics arguments by • Comparing conjectures with previously established results and stated assumptions • Distinguishing correct from flawed logic • Evaluating relationships among evidence and mathematical principles to create generalizations
ELD-MA.9-12 Argue Expressive	Construct mathematics arguments that • Introduce mathematical concept or entity • Share solutions with others • Describe data and/or approach used to solve a problem • State reasoning used to generate own or alternate solutions

Appendix D: Differentiated Instruction

Strategies to accommodate based on student individual needs::

1. Time/General
   a. Extra time for assigned tasks
   b. Adjust length of assignment
   c. Timeline with due dates for reports and projects
   d. Communication system between home and school
   e. Provide lecture notes/outline

2. Processing
   a. Extra Response time
   b. Have students verbalize steps
   c. Repeat, clarify or reword directions
   d. Mini-breaks between tasks
   e. Provide a warning for transitions
   f. Partnering

3. Comprehension
   a. Precise processes for balanced math instructional model
   b. Short manageable tasks
   c. Brief and concrete directions
   d. Provide immediate feedback
   e. Small group instruction
   f. Emphasize multi-sensory learning

4. Recall
   a. Teacher-made checklist
   b. Use visual graphic organizers
   c. Reference resources to
   d. promote independence
   e. Visual and verbal reminders
   f. Graphic organizers

5. Assistive Technology
   a. Computer/whiteboard
   b. Tape recorder
   c. Video Tape

6. Tests/Quizzes/Grading
   a. Extended time
   b. Study guides
   c. Shortened tests
   d. Read directions aloud

7. Behavior/Attention
   a. Consistent daily structured routine
   b. Simple and clear classroom rules
   c. Frequent feedback

8. Organization
   a. Individual daily planner
   b. Display a written agenda
   c. Note-taking assistance
   d. Color code materials

Appendix E: Enrichment

What is the purpose of enrichment?

The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.

- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

Enrichment is ...	**Enrichment is not...**
• Planned and purposeful • Different, or differentiated, work – not just more work • Responsive to students’ needs and situations • A promotion of high-level thinking skills and making connections within content • The ability to apply different or multiple strategies to the content • The ability to synthesize concepts and make real world and cross curricular connections • Elevated contextual complexity • Sometimes independent activities, sometimes direct instruction • Inquiry based or open-ended assignments and projects • Using supplementary materials in addition to the normal range of resources • Choices for students • Tiered/Multi-level activities with flexible groups (may change daily or weekly)	• Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment) • Worksheets that are more of the same (busywork) • Random assignments, games, or puzzles not connected to the content areas or areas of student interest • Extra homework • A package that is the same for everyone • Thinking skills taught in isolation • Unstructured free time

Appendix F: Resources

Textbook: Carter, John, et al, Algebra 1, Glencoe, McGraw-Hill, 2010 (Medford) and 2012 (West)

POR Textbook: Randall, Kennedy, & Hall, Algebra 1, Prentice Hall, Pearson, 2011 (ISBN: 978-0-7854-6917-9)

Appendix G: Climate Change Curriculum Statement

With the adoption of the 2020 New Jersey Student Learning Standards (NJSLS), New Jersey became the first state in the nation to include climate change across content areas. These standards are designed to prepare students to understand how and why climate change happens, the impact it has on our local and global communities and to act in informed and sustainable ways.

Districts are encouraged to utilize the NJSLS to develop interdisciplinary units focused on climate change that include authentic learning experiences, integrate a range of perspectives and are action oriented. While the 2016 NJSLS-English Language Arts (ELA) and Mathematics do not have specific climate change standards, districts may want to consider how they can design interdisciplinary climate change units that incorporate relevant ELA and mathematics standards.

Components of this are tagged throughout the curriculum as appropriate under the “Related Standards” section in each unit.